Workshop on Mathematical Fluid Dynamics and Related Topics

Waseda University

March 10–12, 2020

Place: Room 203, Bldg. 63, Nishi-Waseda Campus

Organizers: Yoshihiro Shibata, Shuichi Kawashima and Hideo Kozono

Workshop on Mathematical Fluid Dynamics and Related Topics

March 10, 2020		March 11, 2020		March 12, 2020	
9:45 – 9:55	registration				
9:55 – 10:00	opening address				
10:00 – 11:20	Series Lec. 1 Robert Denk	10:00 – 11:20	Series Lec. 2 Robert Denk	10:00 – 11:20	Series Lec. 3 Robert Denk
	coffee break		coffee break		coffee break
11:35 – 12:25	Lec. 1 Peer C. Kunstmann	11:35 – 12:25	Lec. 3 Jiang Xu	11:35 – 12:25	Lec. 8 Yasunori Maekawa
12:35 – 13:00	Lec. 2 Motofumi Aoki	12:35 – 13:00	Lec. 4 Ryosuke Nakasato	12:25 – 14:00	Lunch break
		13:00 – 14:30	Lunch break	14:00 – 14:50	Lec. 9 Takahiro Okabe
		14:30 – 15:20	Lec. 5 Yoshiyuki Kagei	15:00 – 15:50	Lec. 10 Takayuki Kubo
		15:30 – 16:20	Lec. 6 Tsukasa Iwabuchi	15:50 – 16:10	coffee break
		16:20 – 16:40	coffee break	16:10 – 17:00	Lec. 11 Yukihito Suzuki
		16:40 – 17:30	Lec. 7 Masashi Ohnawa	17:10 – 18:00	Lec. 12 Takayoshi Ogawa
				18:15 – 20:00	Banquet

March 10 (Tues.)

10:00 – 11:20 Robert Denk (Univ. Konstanz)

Maximal regularity for parabolic evolution equations

Lecture 1: L^p-Sobolev spaces and maximal regularity

Maximal regularity is one of the standard approaches to investigate semilinear and quasilinear parabolic evolution equations. The basic idea of maximal regularity is to find appropriate spaces for the right-hand side and for the solution, where the operator associated to the linearized equation induces an isomorphism. A closed operator $A\colon X \supset D(A) \to X$ acting in a Banach space has maximal L^p-regularity if the Cauchy problem

$$\partial_t u - Au = f \quad (t \in (0,T)), \quad u(0) = u_0$$

has a unique solution $u \in H^1_p((0,T);X) \cap L^p((0,T);D(A))$ depending continuously on the data f and u_0.

In our lectures, we will consider maximal regularity in L^p-Sobolev spaces in time t and space x with $p \in (1,\infty)$. For the initial value $u|_{t=0}$ or for boundary values, one has to describe the trace spaces of L^p-Sobolev spaces. It turns out that these are Besov spaces of non-integer order. If one considers L^p-spaces in time and L^q-spaces in space, so-called Triebel-Lizorkin spaces appear as trace spaces.

[1] R. Denk, M. Hieber, and J. Prüss. $\mathcal{R}$-boundedness, Fourier multipliers and problems of elliptic and parabolic type. *Mem. Amer. Math. Soc.*, 166(788):viii+114, 2003.

[2] P. C. Kunstmann and L. Weis. Maximal L_p-regularity for parabolic equations, Fourier multiplier theorems and H^∞-functional calculus. In *Functional analytic methods for evolution equations*, volume 1855 of *Lecture Notes in Math.*, pages 65–311. Springer, Berlin, 2004.

[3] J. Prüss and G. Simonett. *Moving interfaces and quasilinear parabolic evolution equations*, volume 105 of *Monographs in Mathematics*. Birkhäuser/Springer, [Cham], 2016.

— — — Coffee break — — —

11:35 – 12:25 Peer C. Kunstmann (Karlsruhe Insti, Tech)

H^∞-functional calculus for Stokes operators

We present several results on boundedness of the H^∞-functional calculus for Stokes operators on different types of bounded and unbounded domains. The proofs are partly based on abstract results for square function operators and for Fourier-Bloch multipliers.

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12:35 – 13:00 Motofumi Aoki (Tohoku Univ.)

On regularity of weak solutions of the Navier-Stokes equations in BMO

We consider the regularity of weak solutions of the incompressible Navier–Stokes equations;

$$\begin{cases} \partial_t u - \Delta u + (u \cdot \nabla)u + \nabla p = 0, & t > 0,\ x \in \mathbb{R}^d, \\ \operatorname{div} u = 0, & t > 0,\ x \in \mathbb{R}^d, \\ u\big|_{t=0} = u_0(x), & x \in \mathbb{R}^d, \end{cases}$$

where $u(t,x)$ is the unknown velocity vector; $p(t,x)$ is the unknown pressure; $u_0(x)$ is the given initial velocity vector. Our purpose is to study sufficient condition to obtain regularity of weak solutions. We prove that if $t^{1/2}\|u\|_{BMO}$ is bounded on $(0,T)$, then $t^{\frac{1}{2}}\|\nabla u\|_2$ is bounded on $(0,T]$.

March 11 (Wed.)

10:00 – 11:20 Robert Denk (Univ. Konstanz)

Lecture 2: The concept of $\mathcal{R}$-boundedness and the theorem of Mikhlin

Taking Fourier (or Laplace) transform $\mathscr{F}_t$ in time, one can see that maximal regularity is equivalent to the condition that

$$\mathscr{F}_t^{-1} i\tau(i\tau - A)^{-1}\mathscr{F}_t$$

defines a continuous operator in $L^p(\mathbb{R};X)$. In this case, the (operator-valued) symbol $\tau \mapsto i\tau(i\tau - A)^{-1}$ is said to be an L^p-multiplier.

The theorem of Mikhlin gives a sufficient condition for a symbol to be an L^p-multiplier. In the vector-valued case, one assumes $\mathcal{R}$-boundedness of the symbol ("$\mathcal{R}$" standing for Rademacher or randomized). Several results (e.g., due to Lutz Weis) can be used to prove $\mathcal{R}$-boundedness and, consequently, maximal regularity for operator valued symbols. Here, the Banach space X has to be a UMD space and, in particular, X has to be reflexive.

— — — Coffee break — — —

11:35 – 12:25 Jiang Xu (Nanjing Univ. of Aeronautics and Astronautics)

The optimal time-decay for compressible Navier-Stokes equations in the critical L^p framework

The global existence issue for the isentropic compressible Navier-Stokes equations in the critical regularity framework has been addressed by R. Danchin more than fifteen years ago. However, whether (optimal) time-decay rates could be shown in general critical spaces and any high dimensions has remained an open question. In this talk, we survey recent results not only in the L2 critical framework but also in the more general Lp critical framework, which are based on those collaborative works with R. Danchin and Z. Xin.

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12:35 – 13:00 Ryosuke Nakasato (Tohoku Univ.)

Well-posedness and time-decay estimates for the compressible Hall - magnetohydrodynamic system in L^2 critical framework

We consider the initial value problem for the compressible Hall-magnetohydrodynamic system in the whole space $\mathbb{R}^3$, which is a hyperbolic-parabolic system of conservation laws with non-symmetric diffusion:

$$\begin{cases} \partial_t\widetilde{\rho} + \operatorname{div}(\widetilde{\rho}u) = 0, & t>0, x\in\mathbb{R}^3, \\ \partial_t(\widetilde{\rho}u) + \operatorname{div}(\widetilde{\rho}u\otimes u) + \nabla P(\widetilde{\rho}) = \mathcal{A}u + (\nabla\times\widetilde{B})\times\widetilde{B}, & t>0, x\in\mathbb{R}^3, \\ \partial_t\widetilde{B} - \sigma\Delta\widetilde{B} + \nabla\times\left(\dfrac{(\nabla\times\widetilde{B})\times\widetilde{B}}{\widetilde{\rho}}\right) = \nabla\times(u\times\widetilde{B}), & t>0, x\in\mathbb{R}^3, \\ \operatorname{div}\widetilde{B} = 0, & t>0, x\in\mathbb{R}^3, \\ (\widetilde{\rho}, u, \widetilde{B})|_{t=0} = (\widetilde{\rho}_0, u_0, \widetilde{B}_0), & x\in\mathbb{R}^3, \end{cases} \tag{1}$$

where $\widetilde{\rho}:\mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}_+$, $u:\mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}^3$ and $\widetilde{B}:\mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}^3$ denote the dencity of fluid, the velocity of fluid and the magnetic field, respectively. The pressure $P=P(\widetilde{\rho}):\mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}$ is assumed to be a smooth function of the dencity $\widetilde{\rho}$ and $P'(\widetilde{\rho})>0$, for any $\widetilde{\rho}$. We denote $\mathcal{A}u = \mu\Delta u + (\lambda+\mu)\nabla\operatorname{div} u$; Lamé coefficients μ and λ fulfill the standard parabolicity assumption $\mu>0$ and $\lambda+2\mu>0$; $\sigma>0$ is the magnetic diffusive constant.

We look for a solutions as a perterbation from a constant equilibrium state $(\overline{\rho}, 0, \overline{B})$, where $\overline{\rho}>0$ is a constant density, $0\in\mathbb{R}^3$ is the zero velocity and $\overline{B}\in\mathbb{R}^3$ is a constant magnetic field. In the following, we reformulate the system (1). We introduce $\rho := \widetilde{\rho}-\overline{\rho}$ and $B := \widetilde{B}-\overline{B}$, we see that the system (1) is reformulated as follow:

$$\begin{cases} \partial_t\rho + \overline{\rho}\operatorname{div} u = f & t>0, x\in\mathbb{R}^3, \\ \partial_t u - \dfrac{1}{\overline{\rho}}\mathcal{A}u + \dfrac{P'(\overline{\rho})}{\overline{\rho}}\nabla\rho - \dfrac{1}{\overline{\rho}}(\nabla\times B)\times\overline{B} = g, & t>0, x\in\mathbb{R}^3, \\ \partial_t B - \sigma\Delta B + \dfrac{1}{\overline{\rho}}\nabla\times\big((\nabla\times B)\times\overline{B}\big) - \nabla\times(u\times\overline{B}) = \nabla\times h, & t>0, x\in\mathbb{R}^3, \\ \operatorname{div} B = 0, & t>0, x\in\mathbb{R}^3, \\ (\rho, u, B)|_{t=0} = (\rho_0, u_0, B_0), & x\in\mathbb{R}^3, \end{cases} \tag{2}$$

where f, g, h are the nonlinear terms depending only ρ, u and B.

In this talk, we shall state the results on the well-posedness and time-decay estimates for the solution of (2) in L^2-Besov spaces. This talk is based on the joint work with Profs. Shuichi Kawashima (Waseda University) and Takayoshi Ogawa (Tohoku University).

— — — Lunch Time — — —

14:30 – 15:20 Yoshiyuki Kagei (Tokyo Inst. Tech.)

On the bifurcation and stability of the compressible Taylor vortices

Stability and bifurcation problem is considered for the compressible Navier-Stokes equations in a domain between two concentric cylinders. If the outer cylinder is at rest and the inner one rotates with sufficiently small angular velocity, a laminar flow, called the Couette flow, is stable. When the angular velocity of the inner cylinder increases, beyond a certain value of the angular velocity, the Couette flow becomes unstable and a vortex pattern, called the Taylor vortex, bifurcates and is observed stably. This phenomena is mathematically formulated as a bifurcation and stability problem. In a framework of the incompressible Navier-Stokes equations, the bifurcation and stability of the Taylor vortex has been studied in detail, while, for the compressible Navier-Stokes equations, only the bifurcation of the Taylor vortex has been known, but detailed structure of the bifurcation has remained unknown in many aspects. In this talk, the smoothness of the bifurcation curve and the stability of the compressible Taylor vortex will be shown under axisymmetric perturbations.

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15:30 – 16:20 Tsukasa Iwabuchi (Tohoku Univ.)

Discontinuity of solutions for 2D compressible Navier-Stokes equations

In this talk, we study the continuity of solutions on initial data for compressible Navier-Stokes equations in two space dimensions. The spaces for the continuity has scaling invariant property. As a result, the negative result will be shown for almost all cases of the framework for the temperature with negative derivative regularity.

— — — Coffee break — — —

16:40 – 17:30 Masashi Ohnawa (Tokyo Univ, Marine Sci & Tech.)

On steady solutions with jumps in shallow water equations

In this talk, we study the shallow water equation to understand the dynamics of certain steady flows which could contain discontinuities. The spatial domain under consideration is $\{(x,y) \in \mathbb{R}^2 \mid r = \sqrt{x^2+y^2} \in [r_0, r_1]\}$ for certain positive values r_0 and r_1, and we assume that the flow is axisymmetric without swirling motion. The equation reads

$$(rh)_t + (rhu)_r = 0, \tag{3a}$$

$$(rhu)_t + (rhu^2)_r + rp(h)_r = 0 \quad \text{with} \quad p(h) = gh^2/2, \tag{3b}$$

$$(h,u)(0,r) = (h_0(r), u_0(r)), \tag{3c}$$

where the unknown functions h, u represent depth and radial velocity of the fluid respectively, while the constant g stands for the gravitational acceleration. The flows we have in mind appear when we let tap water imping on plates. The flow diverges radially in a thin layer which abruptly thickens at some point. Based on this observation, we assume that the flow is supersonic at $r = r_0$ and impose boundary conditions of

$$(h,u)(t,r_0) = (h_-, u_-) \quad \text{with} \quad u_-^2 > gh_-. \tag{4}$$

Piecewise smooth steady solutions to (3) and (4) have two types. One is supersonic everywhere, and we do not put boundary conditions at $r = r_1$. The other has a single jump at some point, beyond which the flow transits to subsonic states. In the latter case, the flow is subject to an additional boundary condition at $r = r_1$:

$$h(t,r_1) = h_+. \tag{5}$$

This second type is a simple model of the phenomenon stated above. In real situations, other effects which are neglected in our model such as viscosity, surface roughness, or surface tension could determine the boundary condition at the outlet.

In the first part of the talk, we give conditions for the unique existence of steady solutions. Secondly, we prove the stability of steady solutions based on energy method. Difficulties are twofold; one is in the lack of dissipation of the system and the other is in the treatment of moving interface between supersonic and subsonic states. It turns out that boundary terms do work in favor of stabilization. If time permits, we will also mention results when the boundary condition at the inlet (4) is time dependent.

This work is based on collaboration with Masahiro Suzuki (Nagoya Institute of Technology).

March 12 (Thurs.)

10:00 – 11:20 Robert Denk (Univ. Konstanz)

Lecture 3: Maximal regularity for parabolic boundary value problems

An application of the vector-valued Mikhlin theorem gives maximal regularity for linear parabolic differential operators in the whole space $\mathbb{R}^n$. For boundary value problems in domains, however, one has to study model problems in the half space $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_n > 0\}$ of the form

$$\begin{aligned} \lambda u - Au = f \quad & \text{in } \mathbb{R}^n_+, \\ B_j u = g_j \ (j = 1, \dots, m) \quad & \text{on } \mathbb{R}^{n-1}, \end{aligned}$$

where A is a linear partial differential operator of order $2m$ and $B_1, \dots, B_m$ are boundary operators. We assume the boundary value problem to be parabolic which means, in particular, that the Shapiro-Lopatinskii condition has to be satisfied. Describing the solution operators to parabolic boundary value problems, it is possible to prove $\mathcal{R}$-boundedness.

These results can be applied to quasilinear parabolic problems of the form

$$\partial_t u + A(u)u = F(u) \quad (t > 0), \quad u(0) = u_0.$$

By an application of the contraction mapping principle, one can show that this problem is locally well-posed if the linearized operator $A(u_0)$ has maximal regularity. As an example, one can show well-posedness for the graphical mean curvature flow.

— — — Coffee break — — —

11:35 – 12:25 Yasunori Maekawa (Kyoto Univ.)

Prandtl boundary layer expansion around concave boundary layer

We present a recent progress on the Prandtl boundary layer expansion for the two dimensional Navier-Stokes equations in the half space under the no-slip boundary condition, which is a classical topic in fluid mechanics and important for the analysis of the flow with a small viscosity. It is known by the pioneering work of Sammartino and Caflisch that the Prandtl expansion is valid in a framework of analytic regularity. Recently it is shown by Gerard-Varet, M., and Masmoudi that the Prandtl expansion is valid in a Gevrey 3/2 class around a time-independent concave shear boundary layer, where the key step is the resolvent analysis of the linearized operator that is reduced to the analysis of the 4th order ODE, called the Orr-Sommerfeld equations.

While it has been still open whether Gevrey 3/2 regularity is enough to verify the expansion when the concave shear boundary layer depends on the time variable, or even more generally, when the leading boundary layer is concave but not necessarily a shear type. The main difficulty is that the reduction to the Orr-Sommerfeld equations does not work well in this general case. This talk focuses on a recent progress in this direction (joint work with David Gerard-Varet (Paris 7) and Nader Masmoudi (NYU)).

— — — LUNCH TIME — — —

14:00 – 14:50 Takahiro Okabe (Osaka Univ.)

Annihilation of slow-decay factors of the Navier-Stokes flow by the external force

This article is based on the joint work with Loreonzo Brandolese (Université Lyon 1).

We consider the incompressible Navier-Stokes equations on the whole space $\mathbb{R}^n$, $n \geq 3$. The decay of the energy, i.e., L^2-norm of the velocity, is one of main interests of the mathematical fluid mechanics from the viewpoint of controlling the nonlinear terms.

Over past decades, the algebraic decay rate of the energy decay was investigated. It is known as optimal that $\|u(t)\|_2 \leq C(1+t)^{-(n+2)/4}$, which is an essential decay rate of the nonlinear terms. Later, Fujigaki-Miyakawa (SIAM J. Math. Anal., 2001) derived the leading order terms of the linear part and nonlinear part by using the derivative of the heat kernel. Once leading order terms are revealed, Miyakawa-Schonbek (Math. Bohem., 2001) characterized the necessary and sufficient condition for a rapid energy decay beyond the optimal decay.

Based on this condition, we have to manage

$$\int_0^\infty \int_{\mathbb{R}^n} (u_\ell u_k)(y,s)\, dyds = \delta_{\ell,k}, \quad \ell, k = 1, \ldots, n$$

for rapid decay. For this reason, some symmetry was introduced and a few (initial) flows are discussed to derive a rapid decay.

The aim of this article is to derive a rapid decay for every (small) initial flow with the aid of an associated external force. In other words, for every initial flow we find an external force which annihilate the slow decay factors of the flow. Furthermore, we note that we can take the force which has a compact support in the space-time region.

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15:00 – 15:50 Takayuki Kubo (Ochanomizu Univ.)

Analysis of non-stationary Navier-Stokes equations approximated by the pressure stabilization method

Let us consider the non-stationary Navier-Stokes equations approximated by the pressure stabilization method. As you know, One of the difficulty of analysis for the Navier-Stokes equations is the incompressible condition $\nabla \cdot u = 0$. In order to overcome this difficulty, we often use Helmholtz decomposition. On the other hand, in numerical analysis, some penalty methods are employed. In this talk, we consider the Navier-Stokes equations with incompressible condition approximated by pressure stabilization method, which is one of the penalty method. Namely, we consider the following approximated incompressible condition instead of usual incompressible condition:

$$(u, \nabla\varphi) = \alpha^{-1}(\nabla\pi, \nabla\varphi) \qquad (\forall\varphi \in \widehat{W^1_{q'}}(\Omega)),$$

where α is a perturbation parameter. As $\alpha \to \infty$, the approximated incompressible condition tends to usual incompressible condition formally.

In this talk, we report the local in time existence theorem for solutions to our problem and the error estimate in the L_p in time and the L_q in space framework with $n/2 < q < \infty$ and $\max(1, n/q) < p < \infty$. Moreover we shall report on recent results. This talk are based on some results obtained in our joint work with R. Matsui and H. Kikuchi.

[1] T. Kubo and R. Matsui, On pressure stabilization method for nonstationary Navier-Stokes equations, *Communications on Pure and Applied Analysis*, **17**, No.6, 2283-2307, 2018.

— — — Coffee break — — —

16:10 – 17:00   Yukihito Suzuki   (Waseda Univ.)

On a thermodynamically consistent modeling for complex fluids

In this talk, a thermodynamically consistent modeling for complex fluids will be presented. The modeling is based on a GENERIC formalism [1,2,3], in which reversible processes are modeled as a Hamiltonian system; irreversible processes are modeled as a dissipative system typically with the quadratic dissipation potential; and these two systems are coupled via some degenerate conditions. Complex fluids are modeled introducing additional structural variables called the conformation tensor [3] and Cattaneo heat flux [4], which describe viscoelastic micro-structure and hyperbolic heat conduction, respectively. These structural variables are convected with the fluid flow, and thus their time evolution is naturally represented by the Lie derivative along the flow, whereas the time evolution of other state variables, densities of mass, momentum and entropy, is written in conservation form. Indeed, those densities are quantities to be integrated on a volume, and are connected to conservation laws of mass, momentum and entropy, respectively. In contrast to that, structural variables mentioned above represent the local state of the flow, and nothing to do with any conservation law. These qualitative properties are prescribed by the Poisson bracket in the GENERIC formalism. The barotropic approximation is useful to decouple the mechanics from thermodynamics, and thus make the mathematical analysis easier. It results in a thermodynamically incomplete model. Even in that case, the GENERIC formalism is still applicable and useful. We will present a barotropic model for complex fluids in a GENERIC formalism, in which the mechanical energy of an isolated system never increases. This guarantees not to violate the second law of thermodynamics.

[1] M. Grmela and H.C. Öttinger. Dynamics and thermodynamics of complex fluids. I. Development of a general formalism. *Phys. Rev. E*, **56**, 6620-6632, 1997.

[2] H.C. Öttinger and M. Grmela. Dynamics and thermodynamics of complex fluids. II. Illustrations of a general formalism. *Phys. Rev. E*, **56**, 6633-6655, 1997.

[3] H.C. Öttinger. *Beyond Equilibrium Thermodynamics.* Wiley, 2005.

[4] D.D. Joseph and L. Preziosi. Heat waves. *Rev. Mod. Phys.*, **61**, 41-73, 1989.

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17:10 – 18:00 Takayoshi Ogawa (Tohoku Univ.)

Endpoint maximal regularity for heat equations in the half space with inhomogeneous boundary conditions and sharp trace estimates

Maximal regularity for the initial boundary value problem for the heat equations and higher order parabolic type equations is well established by Weidemaier and Denk-Hieber-Pruss. However the end-point exponent is exceptional case and it is only shown in the case of time L^1 with the homogeneous Besov spaces under the 0-Dirichlet boundary condition by Danchin-Mucha. We consider the initial-boundary value problem for the heat equation in the half Euclidian space with inhomogeneous Dirichlet or Neumann data and show end-point maximal regularity and sharp trace estimate under the setting of homogeneous Besov spaces. This talk is based on the a joint work with Prof. Senjo Shimizu (Kyoto University).

— — — BANQUET (18:15 – 20:00) — — —